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SciVerse ScienceDirect
Journal of Economic Theory 147 (2012) 1313–1330
JOURNAL OF Economic Theory
www.elsevier.com/locate/jet

Introduction to inequality and risk ☆

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Available online 18 May 2012

Abstract

This introduces the symposium on inequality and risk.

JEL classification: D63; D81

Keywords: Inequality; Risk

1. Introduction

The literatures on the measurement of inequality and risk have been inexorably intertwined since the publication of the seminal articles of Atkinson [1] and Rothschild and Stiglitz [36] in the September 1970 issue of the *Journal of Economic Theory*. Both of these articles are concerned with the comparative evaluation of distributions of a single variable. For Atkinson, income

☆ With two exceptions, the articles in this symposium were presented in Paris on June 25–26, 2010 at the ParisTech-*Journal of Economic Theory* Conference on Inequality and Risk to mark the fortieth anniversary of the publication of Atkinson [1] and Rothschild and Stiglitz [36]. The symposium editors are grateful to Alfred Galichon, Jean-Michel Grandmont, Roger Guesnerie, Brian Hill, and Bettina Linsson for their help in organizing this event, to Susan Schulze and Karl Shell for their support of and assistance with the symposium issue, and to the contributors for their comments on a preliminary version of this introduction. They are also grateful to the Collège de France and the Institut Henri Poincaré for providing the conference venues and to AXA University, the AXA Chair "Large Risks in Insurance" (ENSAE–École polytechnique–Université Paris-Dauphine), the Département d'Économie of the École polytechnique, and the Groupement d'intérêt Scientifique "Sciences de la Décision" (ENSAE–HEC–Université Paris X) for their financial support.
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0022-0531/$ – see front matter
http://dx.doi.org/10.1016/j.jet.2012.05.014

distributions are compared in terms of their relative inequality, whereas for Rothschild and Stiglitz, random variables are compared in terms of their relative riskiness. While motivated by different economic problems, their analyses share a common formal structure, which has permitted results obtained using one framework to be used in the other. These two articles have provided the foundation for an extensive literature that has refined, extended, and applied their results.

This symposium illustrates many of the new directions that research on inequality and risk is taking today. In order to place the symposium articles in context, we first provide an overview of the most relevant background results, focusing primarily, but not exclusively, on the contributions of Atkinson [1], Atkinson and Bourguignon [2,3], and Rothschild and Stiglitz [36].[1] In Sections 2 and 3, respectively, we consider the comparative evaluation of inequality and risk for univariate and multivariate distribution functions.[2] In Section 4, we discuss functional forms that have been proposed for summary measures used to evaluate inequality and risk. We then introduce each of the symposium articles in Sections 5, 6, and 7. Eight of these articles extend the stochastic dominance theorems presented in Sections 2 and 3 in novel ways. They are discussed in Section 5. Five articles provide axiomatizations of new functional forms for the analysis of inequality, risk, and welfare. They are discussed in Section 6. The other two articles consider issues related to inequality aversion and risk aversion. They are discussed in Section 7.

2. Univariate comparative evaluations of inequality and risk

Atkinson [1] has considered a number of criteria for regarding one income distribution to exhibit less inequality than a second. Similarly, Rothschild and Stiglitz [36] have considered a number of criteria for regarding one distribution of random variables to be less risky than a second. These criteria are referred to as *dominance criteria*. In each of these articles, the dominance criteria that are considered are shown to be equivalent to each other. By stripping the models of their economic interpretations, it is easy to see that the dominance criteria used in both of these articles are equivalent to each other. This formal equivalence allows us to provide a unified review of the Atkinson and Rothschild–Stiglitz equivalence theorems.

In this section, we consider univariate distributions on $\Omega = [0, \bar{\omega}]$, interpreted as being either distributions of random variables or of income. Distributions are evaluated using the function $W : \Omega \to \mathbb{R}$ that assigns the value

$$W(X) = \int_{\Omega} U(\omega)\, dF(\omega) \tag{1}$$

to the distribution X, where F is the (cumulative) distribution function for X. When X is a random variable, $W(X)$ is the expected utility of a decision-maker who evaluates X using the utility function U. When X is an income distribution, $W(X)$ can be interpreted as being the value assigned to X by a utilitarian social welfare function that uses the utility function U to convert income into an interpersonally-comparable measure of well-being.[3] In this case, the decision-

[1] In focusing our overview in this way, we are neglecting many important contributions to the issues we consider, including earlier antecedents in the economics, mathematics, and statistics literatures to some of the results we discuss.

[2] In the inequality literature, it is more common to say “one-dimensional” and “multidimensional” instead of “univariate” and “multivariate.” In order to provide a unified account of the literatures on inequality and risk, we employ the latter terminology.

[3] Atkinson interprets (1) as an additively separable social welfare function and does not require U to be interpreted as a utility function.

maker can be thought of as being a social planner. The social planner uses the same utility function for all individuals and it need not be anyone's actual utility function.

Let X and Y be two univariate distributions. The corresponding distribution functions are F and G, respectively. Their means are μ_F and μ_G. The Atkinson and Rothschild–Stiglitz dominance criteria all regard distributions with the same distribution function as being equivalent, so we shall define these criteria in terms of distribution functions. In the case of income distributions, this amounts to assuming that all permutations and replications of an income distribution are regarded as being equivalent.

For income distributions with the same mean, F can be regarded as exhibiting no more inequality than G if all inequality-averse social planners weakly prefer F to G. Analogously, for random variables with the same mean, F can be regarded as being weakly less risky than G if all risk-averse expected utility maximizers weakly prefer F to G. These dominance criteria are special cases of the following stochastic dominance relation: F *stochastically dominates* G if

$$\int_{\Omega} U(\omega)\,dF(\omega) \geqslant \int_{\Omega} U(\omega)\,dG(\omega) \quad \text{for all } U \in \mathcal{U}, \tag{2}$$

where $\mathcal{U}$ is a class of real-valued functions on Ω. Different evaluators agree that distributions should be compared using the additively-separable evaluation function in (2), but differ in the functions U that they use to perform the evaluation. This dominance criterion only ranks the distributions being compared if there is unanimous agreement on the part of all evaluators whose U functions are drawn from the class $\mathcal{U}$. Rothschild and Stiglitz considered the class $\mathcal{U}_c$ of all concave functions on Ω and Atkinson considered the smaller class $\mathcal{U}_{ic}$ of all increasing concave functions on Ω. If (2) holds with $\mathcal{U} = \mathcal{U}_{ic}$, then F *second-order stochastic dominates* G.[4]

For random variables, concavity of U corresponds to risk aversion on the part of the decision-maker. In this case, F second-order stochastic dominates G if every expected utility maximizer who is risk averse and who prefers higher values of the outcome ω weakly prefers F to G. For income distributions, concavity of U corresponds to inequality aversion on the part of the social planner. With this interpretation of the model, F second-order stochastic dominates G if every utilitarian social planner who is inequality averse and who regards utility to be increasing in income weakly prefers F to G.

As we have noted, the stochastic dominance criterion used by Rothschild and Stiglitz is

$$\int_{\Omega} U(\omega)\,dF(\omega) \geqslant \int_{\Omega} U(\omega)\,dG(\omega) \quad \text{for all } U \in \mathcal{U}_c. \tag{3}$$

In order for (3) to be satisfied, F and G must have the same mean. It is well known that (3) is equivalent to

$$\int_{\Omega} U(\omega)\,dF(\omega) \geqslant \int_{\Omega} U(\omega)\,dG(\omega) \quad \text{for all } U \in \mathcal{U}_{ic} \text{ and } \mu_F = \mu_G. \tag{4}$$

See, for example, Müller and Stoyan [34, Theorem 1.5.3]. Thus, Rothschild and Stiglitz are comparing distributions according to second-order stochastic dominance. For fixed mean comparisons, (4) is the stochastic dominance criterion used by Atkinson.

Rothschild and Stiglitz have shown that (3) is equivalent to the following *integral conditions*:

[4] If $\mathcal{U}$ is the set of all increasing functions on Ω, then (2) defines *first-order stochastic dominance*.

$$\int_0^{\omega} F(t)\,dt \leqslant \int_0^{\omega} G(t)\,dt \quad \text{for all } \omega \in [0, \bar{\omega}) \text{ and } \mu_F = \mu_G. \tag{5}$$

Recall that the Lorenz curve at $t \in [0, 1]$ plots the proportion of the total amount of ω associated with the bottom $t\%$ of the values of ω (measured as a fraction of 1). Let L_F and L_G denote the Lorenz curves for F and G, respectively. The distribution function F *weakly Lorenz dominates* the distribution function G if the Lorenz curve for F lies nowhere below the Lorenz curve for G. Atkinson has shown that the integral conditions (5) are equivalent to requiring that F weakly Lorenz dominates G and that F and G have the same mean. Formally,

$$L_F(t) \geqslant L_G(t) \quad \text{for all } t \in [0, 1] \text{ and } \mu_F = \mu_G. \tag{6}$$

Thus, for equal mean comparisons, defining "less risky" or "less unequal" using Lorenz dominance is equivalent to defining these concepts in terms of second-order stochastic dominance.

Informally, a mean-preserving spread of a random variable takes mass from the center of the distribution and adds it to the tails in such a way that the mean is unaffected. Rothschild and Stiglitz provided formal definitions of a mean-preserving spread for distributions that are either discrete or have densities. They showed that if G is obtained from F by a sequence of mean-preserving spreads, then the integral conditions (5) are satisfied. However, they were only able to establish a partial converse to this result: if F and G satisfy the integral conditions, then G can be obtained from F to an arbitrary degree of approximation by a sequence of mean-preserving spreads. More precisely, if F and G satisfy the integral conditions, then there exist two sequences of discrete distributions F_n and G_n converging to F and G respectively such that for each n, G_n differs from F_n by a finite number of mean-preserving spreads.

An exact equivalence result for arbitrary distribution functions on Ω can be obtained using the more general definition of a mean-preserving spread due to Machina and Pratt [28]. With their definition, G differs from F by a *mean-preserving spread* if $\mu_F = \mu_G$ and there exist $\omega', \omega'' \in \Omega$ with $\omega' \leqslant \omega''$ such that (a) G has at least as much mass as F on every open subinterval of $[0, \omega')$ and on every open subinterval of $(\omega'', \bar{\omega}]$ and (b) G has no more mass than F on every open subinterval of (ω', ω''). Note that with this definition, F differs from itself by a mean-preserving spread. Conditions (a) and (b) are equivalent to requiring that (a′) $G - F$ is increasing on $[0, \omega')$ and on $(\omega'', \bar{\omega}]$ and (b′) $G - F$ is decreasing on (ω', ω''). See Müller and Stoyan [34, p. 28]. The integral conditions (5) are equivalent to:

$$G \text{ can be obtained from } F \text{ by a sequence of mean-preserving spreads.} \tag{7}$$

Thus, if (7) is used to define what it means for F to be less risky than G, then this concept of "less riskiness" is equivalent to those defined using second-order stochastic dominance and Lorenz dominance.

The distribution function F differs from the distribution function G by an *equalizing transfer* if and only if G differs from F by a mean-preserving spread. Thus, (7) can be restated as:

$$F \text{ can be obtained from } G \text{ by a sequence of equalizing transfers.} \tag{8}$$

In the income inequality literature, including in Atkinson's article, equalizing transfers, not mean-preserving spreads (disequalizing transfers), are used as a criterion for determining when one distribution function is no more unequal than a second.

For distributions of income for a finite population, a *Pigou–Dalton transfer* is a transfer of income from a richer to a poorer person that does not exceed the difference in their initial incomes.

By allowing the size of the transfer to be either zero or the difference in the two incomes, this definition includes the cases of a null transfer and a permutation of the two incomes. A Pigou–Dalton transfer is an equalizing transfer. Hardy et al. [21, Lemma 2, p. 47] have shown that for two distributions X and Y with the same means, X weakly Lorenz dominates Y (i.e., F weakly Lorenz dominates G) if and only if X can be obtained from Y by a finite number of Pigou–Dalton transfers.

Rothschild and Stiglitz also considered regarding one random variable to be at least as risky as a second if the former can be obtained from the latter by adding zero-conditional-mean noise. In terms of the distribution functions F and G, G is *equal to F plus noise* if there exists a pair of jointly distributed random variables (X, Z) on $\Omega \times [-\bar{\omega}, \bar{\omega}]$ with $E[Z \mid \omega] = 0$ for all $\omega \in \Omega$ such that F and G are the distribution functions of X and $X + Z$, respectively. Note that if G is equal to F plus noise, then F and G have the same mean. The distribution function F can be regarded as being weakly less risky than the distribution function G if

$$G \text{ is equal to } F \text{ plus noise.} \tag{9}$$

This concept of "less riskiness" is equivalent to those defined using second-order stochastic dominance, Lorenz dominance, and mean-preserving spreads.

To summarize, we have seen that (3), (4), (5), (6), (7), (8), and (9) are all equivalent dominance criteria for comparing distribution functions with common means. In defining the last three of these dominance conditions, appeal is made to the existence of sequences of distributions with certain properties. Explicit constructions of these sequences have been provided by Rothschild and Stiglitz and by Machina and Pratt, thereby permitting these criteria to be implemented in practice.

We have only considered comparing a single pair of distribution functions. On the set of all distribution functions with a given mean, these dominance criteria generate a partial order (i.e., a reflexive, transitive, and antisymmetric binary relation) on this set.[5]

Atkinson also briefly considered variable-mean comparisons. In this case, his stochastic dominance condition is

$$\int_{\Omega} U(\omega)\, dF(\omega) \geqslant \int_{\Omega} U(\omega)\, dG(\omega) \quad \text{for all } U \in \mathcal{U}_{ic}, \tag{10}$$

which is simply (4) without the requirement that $\mu_F = \mu_G$. He notes that (10) implies that $\mu_F \geqslant \mu_G$. Conversely, if both $\mu_F \geqslant \mu_G$ and F weakly Lorenz dominates G, then (10) holds. An exact equivalence can be obtained using generalized Lorenz curves. For all $t \in [0, 1]$, the value of the *generalized Lorenz curve* for the distribution F is

$$GL_F(t) = \mu_F L_F(t). \tag{11}$$

The distribution F *weakly generalized Lorenz dominates* the distribution G if

$$GL_F(t) \geqslant GL_G(t) \quad \text{for all } t \in [0, 1]. \tag{12}$$

This dominance criterion is equivalent to (10). See, for example, Shorrocks [39, p. 6].

5 There are other dominance criteria that are equivalent to the ones considered here. See Marshall et al. [29] and Müller and Stoyan [34].

3. Multivariate comparative evaluations of inequality and risk

The economics literature on multivariate inequality and risk builds on the seminal articles of Atkinson and Bourguignon [2,3] and Kolm [25]. For multivariate distributions, less inequality or risk may be associated with less dispersion (as in the univariate case) or with less positive dependence between the variables. Kolm focused primarily on the first of these phenomenon, whereas Atkinson and Bourguignon took both into account, but focused on the second. Atkinson and Bourguignon restricted attention to the bivariate case, as has much of the subsequent economics literature that considers interdependence between the variables. In Atkinson and Bourguignon [2], both variables have symmetric roles, but they are treated asymmetrically in Atkinson and Bourguignon [3].[6]

Let $X = (X_1, \dots, X_d)$ and $Y = (Y_1, \dots, Y_d)$ be multivariate distributions on $\Omega = \Omega_1 \times \cdots \times \Omega_d$, where $\Omega_i = [0, \bar{\omega}_i]$, $i = 1, \dots, d$. In the inequality context, these variables are indicators of well-being, such as income and health status. In the case of risk, the variables are typically monetary returns from risky activities that are not perfectly substitutable, such as investing in different financial assets. The corresponding joint (cumulative) distribution functions are F and G, respectively. Let $\bar{F}$ and $\bar{G}$ denote the corresponding survival functions. When $d = 1$, $\bar{F} = 1 - F$ and $\bar{G} = 1 - G$, but, in general, these equalities do not hold for $d \geqslant 2$. The marginal distribution functions of X_i and Y_i are denoted by F_i and G_i. A *Fréchet class* is a set of joint distribution functions with the same marginals. Letting $\omega = (\omega_1, \dots, \omega_d)$ and defining Ω as above, the evaluation function W in (1) and the stochastic dominance criterion in (2) can be applied to multivariate distributions. As in Section 2, W can be interpreted as being a utilitarian social welfare function or as an expected utility functional.

For bivariate distributions, Atkinson and Bourguignon [2] analyzed first-order and second-order stochastic dominance when the variables are either substitutes or complements. We restrict attention to substitutable variables. Atkinson and Bourguignon assumed that the class of functions $\mathcal{U}$ used for their stochastic dominance comparisons are continuously differentiable to any required degree and that the distribution functions F and G have densities, which we denote by f and g.

Atkinson and Bourguignon's criterion for F to first-order stochastic dominate G when $d = 2$ is that (2) holds for all U in

$$\mathcal{U}_{\mathrm{AB}_1} = \{U \in \mathcal{U} \mid U_1, U_2 \geqslant 0;\ U_{12} \leqslant 0\}.$$

The restrictions on the first partials of U require U to be nondecreasing in each variable. The nonpositive cross-partial restriction captures the assumption that the variables are substitutes: the marginal contribution of one variable is nonincreasing in the value of the other variable.

For $x, y \in \mathbb{R}^d$, let $x \vee y = (\max\{x_1, y_1\}, \dots, \max\{x_d, y_d\})$ and $x \wedge y = (\min\{x_1, y_1\}, \dots, \min\{x_d, y_d\})$. The real-valued function U on Ω is *submodular* (or *L-subadditive*) if

$$U(x) + U(y) \geqslant U(x \vee y) + U(x \wedge y) \quad \text{for all } x, y \in \Omega. \tag{13}$$

If U is twice differentiable, then U is submodular if and only if $U_{ij}(x) \leqslant 0$ for all $i, j \in \{1, \dots, d\}$ and all $x \in \Omega$. See Müller and Stoyan [34, Theorem 3.9.3]. *F dominates G according to the nondecreasing submodular order* if (2) holds with $\mathcal{U}$ chosen to be the class of all nondecreasing submodular functions. In the bivariate case, this is the class $\mathcal{U}_{\mathrm{AB}_1}$.

[6] See Joe [23] and Müller and Stoyan [34] for detailed discussions of dependence concepts and multivariate stochastic orders. Trannoy [42] provides a good introduction to multivariate inequality dominance.

F dominates G according to the lower-orthant order if

$$F(\omega) \leqslant G(\omega) \quad \text{for all } \omega \in \Omega \tag{14}$$

and *F dominates G according to the upper-orthant order* if

$$\bar{F}(\omega) \leqslant \bar{G}(\omega) \quad \text{for all } \omega \in \Omega. \tag{15}$$

If both (14) and (15) hold, then *F dominates G according to the concordance order*, in which case G is said to be *more concordant than F*. If G is more concordant than F, then F and G have the same marginals. See Meyer and Strulovici [30, ft. 23].

In the bivariate case, Atkinson and Bourguignon have shown that (a) F dominates G according to the nondecreasing submodular order if and only if (b) F dominates G according to the lower-orthant order.[7] Moreover, it follows from Müller and Stoyan [34, Theorem 3.8.2] that if F and G are bivariate and belong to the same Fréchet class (i.e., they have the same marginal distribution functions), then the nondecreasing submodular order, the lower-orthant order, the upper-orthant order, and the concordance order are equivalent.[8] As we have noted, if F and G are comparable according to the concordance order, then they belong to the same Fréchet class.

For discrete bivariate distributions X and Y, the distribution function *G is obtained from F by a positive* 2*-rearrangement* if there exist ω_i' and ω_i'' with $\omega_i' < \omega_i''$ for $i = 1, 2$ and $\epsilon > 0$ such that

$$g(\omega_1, \omega_2) = \begin{cases} f(\omega_1, \omega_2) + \epsilon, & \text{if } (\omega_1, \omega_2) = (\omega_1', \omega_2') \text{ or } (\omega_1'', \omega_2''); \\ f(\omega_1, \omega_2) - \epsilon, & \text{if } (\omega_1, \omega_2) = (\omega_1'', \omega_2') \text{ or } (\omega_1', \omega_2''); \\ f(\omega_1, \omega_2), & \text{otherwise.} \end{cases} \tag{16}$$

Positive 2-rearrangements were introduced by Hamada [20]. Such a rearrangement transfers mass from the off-diagonal to the diagonal corners of the rectangle defined by the points (ω_1', ω_2'), (ω_1'', ω_2'), (ω_1', ω_2''), and (ω_1'', ω_2''). This kind of transfer leaves the marginal distributions unchanged, but increases the correlation between the two variables. Epstein and Tanny [11, Theorem 1] and Tchen [41, Theorem 1] have shown that if F and G belong to the same Fréchet class, then G is more concordant than F if and only if G can be obtained from F by a finite sequence of positive 2-rearrangements.

The five partial orderings of bivariate distribution functions obtained using the nondecreasing submodular order, the lower-orthant order, the upper-orthant order, the concordance order, and sequences of positive 2-rearrangements provide alternative ways of assessing the positive dependence between distribution functions. In general, these partial orderings do not coincide, but, as we have seen, they are equivalent when the distributions are discrete and belong to the same Fréchet class.

Atkinson and Bourguignon's criterion for F to second-order stochastic dominate G when F and G are bivariate is that (2) holds for all U in

$$\mathcal{U}_{\mathrm{AB}_2} = \{U \in \mathcal{U} \mid U_1, U_2 \geqslant 0;\ U_{12} \leqslant 0;\ U_{11}, U_{22} \leqslant 0;\ U_{112}, U_{221} \geqslant 0;\ U_{1122} \leqslant 0\}.$$

The restrictions on the second-order own partials require the social welfare function to exhibit inequality aversion with respect to each variable taken separately. Hence, equalizing transfers

[7] As they note, the sufficiency of (b) for (a) was established by Hadar and Russell [19, Theorem 5.7].

[8] If $d = 2$, then $\bar{F}(x_1, x_2) = 1 - F_1(x_1) - F_2(x_2) + F(x_1, x_2)$, which is why the lower- and upper-orthant orders are equivalent for bivariate distributions that belong to the same Fréchet class. For $d > 2$, the survival function cannot be determined from the distribution function and the marginals.

of either variable are welfare improving. Moyes [31,32] has identified the welfare-improving transformations that correspond to the restrictions on the third and fourth derivatives.

For bivariate distributions, Atkinson and Bourguignon have shown that necessary and sufficient conditions for F to stochastically dominate G for the class of utility functions $\mathcal{U}_{\mathrm{AB}_2}$ are that

$$\int_0^{\omega_i} F_i(t)\,dt \leqslant \int_0^{\omega_i} G_i(t)\,dt \quad \text{for all } \omega_i \in \Omega_i,\ i = 1, 2, \tag{17}$$

and

$$\int_0^{\omega_1}\int_0^{\omega_2} F(s,t)\,ds\,dt \leqslant \int_0^{\omega_1}\int_0^{\omega_2} G(s,t)\,ds\,dt \quad \text{for all } \omega \in \Omega. \tag{18}$$

These conditions are multivariate analogues of the univariate integral conditions in (5) but without any restrictions on the means of the variables.[9] In (17), the integrals are applied to the marginal distributions of each variable, whereas in (18), they are applied to the joint distribution functions. If F and G belong to the same Fréchet class, then (17) is automatically satisfied because the two distributions have the same marginals.

In Atkinson and Bourguignon [3], distributions of two variables are also considered, but now they are treated asymmetrically. The first variable is income and the second is some measure of need. They assume that the measure of need can take on a fixed finite number of values, which are indexed by $j = 1, \ldots, m$ in decreasing order of need. They further assume that the distributions being compared all have the same marginals for the need variable. Let p_j, $j = 1, \ldots, m$, be the proportion of the population in the jth of the need categories. As in the univariate case, incomes are distributed on $\Omega = [0, \bar{\omega}]$. Now let $X = (X_1, \ldots, X_m)$ and $Y = (Y_1, \ldots, Y_m)$ be two distributions of income on Ω, where the jth component is the distribution for the jth need subgroup. The corresponding (cumulative) distribution functions are $F = (F_1, \ldots, F_m)$ and $G = (G_1, \ldots, G_m)$, respectively. Let $U = (U_1, \ldots, U_m)$ be the m-tuple of utility functions for income of the m subgroups. In this setting, the stochastic dominance condition (2) can be rewritten as

$$\sum_{j=1}^{m} p_j \int_{\Omega} U_j(\omega)\,dF_j(\omega) \geqslant \sum_{j=1}^{m} p_j \int_{\Omega} U_j(\omega)\,dG_j(\omega) \quad \text{for all } U \in \mathcal{U}. \tag{19}$$

The requirement that subgroups with higher indices are less needy is formalized by requiring that

$$U_j'(\omega) - U_{j+1}'(\omega) \geqslant 0 \quad \text{for all } \omega \in \Omega,\ j = 1, \ldots, m-1. \tag{20}$$

For their second-order dominance result, Atkinson and Bourguignon further assume that

$$U_j''(\omega) - U_{j+1}''(\omega) \leqslant 0 \quad \text{for all } \omega \in \Omega,\ j = 1, \ldots, m-1. \tag{21}$$

In other words, (20) requires that the marginal utility of income $U_j'(\omega)$ is nondecreasing in need for any income ω, with (21) additionally requiring that the marginal utility difference $U_j'(\omega) - U_{j+1}'(\omega)$ is nondecreasing in income.

[9] They can also be viewed as being the multivariate counterparts of the generalized Lorenz dominance conditions in (12).

Atkinson and Bourguignon's criteria for F to *first-order* and *second-order stochastic dominate G in incomes* are that (19) is satisfied for all U in

$$\mathcal{U}_{\mathrm{AB}_3} = \{U \in \mathcal{U} \mid \text{(20) holds and } U_j' \geqslant 0,\ j = 1, \ldots, m\}$$

and

$$\mathcal{U}_{\mathrm{AB}_4} = \{U \in \mathcal{U} \mid \text{(20) and (21) hold and } U_j' \geqslant 0, U_j'' \leqslant 0,\ j = 1, \ldots, m\},$$

respectively. In addition to (20), $\mathcal{U}_{\mathrm{AB}_3}$ only requires utility to be nondecreasing in income for each subgroup, whereas $\mathcal{U}_{\mathrm{AB}_4}$ additionally requires the social planner to be inequality averse for each need subgroup and for (21) to hold. For both of these classes of utility functions, it is welfare improving to transfer income from a less needy person to someone who is more needy if they have the same initial incomes. With the second class, it is also welfare improving to make a transfer of income from a richer to a poorer person if they are equally needy.

Atkinson and Bourguignon have shown that their first-order and second-order stochastic dominance criteria for incomes are equivalent to dominance criteria that are applied sequentially to the different needs subgroups.[10] A necessary and sufficient condition for (19) to hold for all U in $\mathcal{U}_{\mathrm{AB}_3}$ is that

$$\sum_{j=1}^{k} p_j F_j(\omega) \leqslant \sum_{j=1}^{k} p_j G_j(\omega) \quad \text{for all } \omega \in \Omega,\ k = 1, \ldots, m, \tag{22}$$

whereas for all U in $\mathcal{U}_{\mathrm{AB}_4}$,

$$\sum_{j=1}^{k} p_j \int_0^{\omega} F_j(t)\,dt \leqslant \sum_{j=1}^{k} p_j \int_0^{\omega} G_j(t)\,dt \quad \text{for all } \omega \in \Omega,\ k = 1, \ldots, m, \tag{23}$$

is necessary and sufficient. In (22), starting with the neediest group, we sequentially add the next neediest group's incomes until all of the groups have been considered and require the resulting distribution for F to first-order stochastic dominate that for G in each step in this sequence. In (23), in each step, the resulting distribution for F is required to generalized Lorenz dominate that for G. For this reason, the latter criterion is known as *sequential generalized Lorenz dominance*. In the last step, these criteria are simply the univariate first-order stochastic dominance and generalized Lorenz criteria applied to all incomes.

4. Functional forms

In addition to analyzing inequality dominance criteria, Atkinson [1] proposed a method for constructing a summary index of inequality from a social welfare function and used this construction to develop what is now known as the Atkinson class of inequality indices. It is for these contributions that Atkinson's article is best known today.

Atkinson used the function W in (1) as a social welfare function. The *equally-distributed-equivalent income* associated with X for the social welfare function W is the income $\Xi_W(X)$ defined implicitly by

[10] For a good introduction to sequential dominance criteria, see Lambert [26, Section 3.6].

$$\int_{\Omega} U\big(\Xi_W(X)\big)\, dF(\omega) = \int_{\Omega} U(\omega)\, dF(\omega). \tag{24}$$

If everybody had the income $\Xi_W(X)$, then the level of social welfare would be the same as with X. $\Xi_W(X)$ is the social welfare analogue of the certainty equivalent in the theory of risky decision-making. Using the function Ξ_W, Atkinson defined the inequality index $I_W : \Omega \to \mathbb{R}$ associated with W by setting

$$I_W(X) = 1 - \frac{\Xi_W(X)}{\mu(X)}, \tag{25}$$

where $\mu(X)$ is the mean of X. $I_W(X)$ measures the fraction of total income that could be destroyed without affecting the level of social welfare if incomes were equalized. Provided that $U \in \mathcal{U}_{ic}$, this index takes on values between 0 and 1, attaining its minimum when incomes are equally distributed.

The procedure used in (25) to construct an inequality index from a social welfare function can be used even if W does not take the particular form given in (1) provided that the equally-distributed-equivalent income function is well defined. This procedure was independently proposed by Kolm [24] and later popularized by Sen [38], so an inequality index constructed in this way is called an *Atkinson–Kolm–Sen inequality index*.

A *relative index of inequality* is one that is invariant to a proportional scaling of all incomes. For I_W to be a relative index of inequality, Ξ_W must be homogeneous of degree 1 or, equivalently, W must be homothetic. As Atkinson has noted, if $U \in \mathcal{U}_{ic}$, then this requirement is satisfied if and only if Ξ_W has *the mean of order r* form given by:

$$\Xi_W(X) = \begin{cases} [\int_{\Omega} \omega^r\, dF(\omega)]^{1/r}, & \text{if } r \leqslant 1 \text{ and } r \neq 0; \\ \exp[\int_{\Omega} \ln(\omega)\, dF(\omega)], & \text{if } r = 0, \end{cases} \tag{26}$$

where $r \leqslant 1$ is a scalar that is inversely related to the degree of inequality aversion. For a finite population of n individuals with income distribution $x = (x_1, \ldots, x_n)$, (26) may be rewritten as:

$$\Xi_W(x) = \begin{cases} [\frac{1}{n} \sum_{i=1}^n x_i^r]^{1/r}, & \text{if } r \leqslant 1 \text{ and } r \neq 0; \\ \prod_{i=1}^n x_i^{1/n}, & \text{if } r = 0. \end{cases} \tag{27}$$

An inequality index of the form obtained by substituting (26) or (27) into (25) is an *Atkinson inequality index*.

The generalized Gini inequality indices introduced by Weymark [43] are defined using an equally-distributed-equivalent income function that is a weighted sum of rank-ordered incomes. For an income distribution $x = (x_1, \ldots, x_n)$, let $\tilde{x}$ denote a permutation of x in which the components of x have been ordered in a nonincreasing way. The equally-distributed-equivalent income function for a *generalized Gini inequality index* is given by

$$\Xi_W(x) = \sum_{i=1}^n a_i \tilde{x}_i \tag{28}$$

for all x in the domain of W, where $a_i \geqslant 0$ for all $i = 1, \ldots, n$. Homogeneity of degree 1 of Ξ_W requires that $\sum_{i=1}^n a_i = 1$. Inequality aversion is equivalent to requiring the income weights in (28) to be nondecreasing. By setting $a_i = (2i-1)/n^2$ and substituting (28) into (25), we obtain the Gini index of relative inequality. By reinterpreting x as the vector of state-dependent returns for a discrete random variable, (28) is the *certainty-equivalent* function for a decision-maker who uses the discrete version of the *rank-dependent expected utility* function introduced by Yaari [45].

For a continuous distribution X on Ω, the analogue of (28) is:

$$\Xi_W(X) = \int_{\Omega} \phi\big(1 - F(\omega)\big)\, dF(\omega), \tag{29}$$

where $\phi : [0, 1] \to \mathbb{R}$ is continuous and nondecreasing. If ϕ is differentiable, then (29) may be rewritten as:

$$\Xi_W(X) = \int_{\Omega} \phi'(1 - t) Q_X(t)\, dt, \tag{30}$$

where

$$Q_X(t) = \inf_{x \in \mathbb{R}} \big\{F(x) > t\big\} \tag{31}$$

is the *quantile function* for the distribution X. See Galichon and Henry [15]. The values of $\phi'(1 - t)$ correspond to the weights in (28). Inequality or risk aversion is equivalent to requiring ϕ to be concave.

These rank-dependent objective functions exhibit two important properties. First, they preserve first-order stochastic dominance, so higher outcomes are preferred to smaller ones. Second, if the decision-maker is indifferent between comonotonic income distributions or vectors of returns, then he is also indifferent between convex combinations of them, a property known as *comonotonic independence*.[11]

5. New directions in stochastic dominance analysis

The first eight contributions to the symposium develop new stochastic dominance equivalence theorems. The first two consider univariate stochastic dominance and the other six consider multivariate stochastic dominance.

Chakravarty and Zoli [7] suppose that the outcome variables only take on nonnegative integer values from 0 to m. This restriction is natural in applications such as comparing distributions of indices of social exclusion, health status, and literacy. For a fixed, finite population, they consider distributions x and y whose means need not be equal and establish the equivalence of five dominance conditions. Of particular note is that they show that x weakly generalized Lorenz dominates y if and only if x is weakly preferred to y using any inequality-averse generalized Gini welfare measure or, equivalently, if $\tilde{x}$ can be obtained from $\tilde{y}$ by applying a sequence of rank-preserving transformations that move each position's outcome closer to its final value. Explicit constructions of these sequences are provided.

Le Breton et al. [27] consider income distribution functions for two subgroups (e.g., men and women) and investigate the extent to which one of these groups is discriminated against relative to the other. To do this, they construct first-order and second-order discrimination curves that show the extent to which the comparison distribution function F_c is skewed towards lower incomes relative to the reference distribution function F_r. For fixed F_r, it is shown that the

[11] The vectors ω' and ω'' are *comonotonic* if $(\omega'_i - \omega'_j)(\omega''_i - \omega''_j) \geqslant 0$ for all i, j.

second-order discrimination curve for F_c lies nowhere below that of G_c (i.e., F_c exhibits less second-order discrimination than G_c) if and only if for every income x, there is less discrimination with F_c than with G_c according to the Gastwirth [16] discrimination index truncated at x. When F_r is the uniform distribution, this equivalence reduces to that between the stochastic dominance condition (10) and generalized Lorenz dominance (12).

Moyes [32] is a slightly revised English translation of Moyes [31]. The latter article has played an influential role in the development of multivariate inequality dominance criteria by French-speaking economists, but it is little known outside that community. Ostensibly, the purpose of this article is to extend the equivalence theorems of Atkinson and Bourguignon [3] to allow for the marginal distributions of needs to have different means in the distributions being compared, but it does much more than that. In their informal discussion, Atkinson and Bourguignon [2,3] have related the restrictions they have imposed on their classes of utility functions to various transfer principles, as we have done in our discussion of the classes $\mathcal{U}_{\mathrm{AB}_3}$ and $\mathcal{U}_{\mathrm{AB}_4}$. Moyes explicitly identifies the welfare-improving transformations for bivariate distributions that correspond to the restrictions imposed on the utility functions by Atkinson and Bourguignon in their two articles. He does this both for the case in which the needs variable is ordinal and for the case in which it is cardinal. These transformations apply to one or both of the two variables and take a number of forms—augmentations, permutations, single transfers, and composite transfers.

Gravel and Moyes [18] further explore the case considered by Moyes [31,32] in which one of the two variables is cardinal and transferable (e.g., income) and the other is ordinal and nontransferable (e.g., health status). Their objective is to establish an equivalence between the partial orders generated by (a) the utilitarian unanimity criteria for a given class of utility functions, (b) sequences of elementary transformations that are welfare improving, and (c) implementable criteria that allow one to identify what distributions can be ordered. In the univariate case, Lorenz dominance is an example of what they mean by an implementable criterion. In their main result, Gravel and Moyes identify the class of utility functions and set of transformations needed to establish this kind of equivalence theorem when implementability is defined using the ordered poverty-gap criterion introduced by Bourguignon [6]. With this criterion, a weighted sum of the aggregate income shortfalls from the poverty line for each needs category is used to compare distributions, where the weights are the marginal densities of each needs group and the poverty lines are needs-group specific. Two distributions are only ordered if this comparison is invariant to the poverty lines used subject to the proviso that they are nondecreasing in need.

For bivariate distributions, Muller and Trannoy [35] develop sufficient conditions for second-order stochastic welfare dominance comparisons using a compensation approach that supposes that one continuous, cardinally-measurable variable (e.g., income) can be used to make transfers in order to compensate for unfavorable realizations of the needs variable (e.g., health status), which is also assumed to be cardinally measurable, but can be either discrete or continuous. In terms of their terminology, the former is the compensating attribute of well-being and the latter is the compensated attribute. They do not require the marginal distributions of the needs variable to be fixed. The compensation principles that Muller and Trannoy consider are formalized as restrictions on the classes of utility functions used in the utilitarian unanimity criterion. Although, by construction, the two variables play asymmetric roles, by allowing the needs variable to be continuous, Muller and Trannoy are able to consider restrictions on the utility functions of the kind employed by Atkinson and Bourguignon [3] in the framework used by Atkinson and Bourguignon [2], thereby integrating the two approaches. Of particular note is that Muller and Trannoy have generalized the sequential generalized Lorenz dominance criterion to allow for continuous distributions of the needs variable.

Decancq [10] develops multivariate generalizations of the bivariate equivalences for positive 2-rearrangements when the distributions functions being compared are discrete and belong to the same Fréchet class. Suppose that ω' and ω'' are two d-vectors with $\omega' \leqslant \omega''$ that differ in $k \geqslant 2$ components. The set of all 2^k combinations of the components of these two vectors define the vertices of a k-dimensional hyperbox. A vertex in this hyperbox is odd if the number of components in which it differs from ω' is odd; otherwise it is even. Decancq considers two different ways of shifting mass between even and odd vertices, both of which coincide with a positive 2-rearrangement when $k = 2$. In one case, the ability to obtain distribution G from distribution F using a sequence of such rearrangements is equivalent to G dominating F according to the lower-orthant order; whereas with the other, the equivalence is with the upper-orthant order. The ability of obtaining G from F using either kind of sequence is equivalent to the concordance order.

Meyer and Strulovici [30] also consider dependence orders for multivariate distributions. As they note, for bivariate distributions, however one defines positive dependence, the distributions X and Y are positively dependent if and only if $-X$ and Y are negatively dependent. This symmetry breaks down in higher dimensions. As a consequence, the analysis of dependence concepts when there are more than two dimensions is inherently more difficult than in the bivariate case. Symmetry breaking is also present in Decancq's article, with one of his kinds of rearrangements resulting in more positive dependence when k is even and less when k is odd.

In addition to the supermodular order and the concordance order, Meyer and Strulovici consider three new partial orders: *greater weak association*, the *convex-modular order*, and the *dispersion order*. The first considers every possible partition of the variables into two subsets and compares distributions in terms of the correlation between scalar aggregates of these subsets. The second compares distributions using the Rothschild and Stiglitz [36] concept of riskiness applied to additively separable scalar aggregates of all of the variables. The third compares distributions in terms of the dispersion of the distribution functions of their order statistics, which turns out to be equivalent to comparing riskiness in the Rothschild–Stiglitz sense of the sum of indicator functions at each point in the support of the distributions. For bivariate distributions, these five orders coincide, whereas for four or more dimensions, they are strictly ranked. For three dimensions, four are strictly ranked and two are equivalent.

In the univariate case, the stochastic dominance criterion (3) used by Rothschild and Stiglitz [36] identifies distribution X as being more risky than Y if the latter is preferred by all expected utility maximizers who are risk averse, that is, who have concave utility functions. As we have seen, this is equivalent to saying that X can be obtained from Y by a sequence of mean-preserving spreads or by adding noise to the latter distribution. Müller and Scarsini [33] note that the addition of noise introduces the possibility that the decision-maker ends up with less than his initial wealth and, by the univariate equivalence theorem, this *fear of loss* is equivalent to the concavity restriction on the utility function. With multivariate distributions, it is possible to add zero-conditional-mean noise without ever having a loss in all dimensions. In their contribution to this symposium, Müller and Scarsini argue that in the multivariate case, this fear of loss is best captured by restricting attention to aversion to the addition of zero-conditional-mean noise that results in possible gains or losses in all variables. This restriction is captured by applying the multivariate version of the stochastic dominance criterion in (2) to the class of what are known as *inframodular* utility functions. Müller and Scarsini identify a multivariate generalization of a mean-preserving transfer called an *inframodular transfer* and show that all expected utility maximizers with an inframodular utility function prefer distribution X to Y if and only if the former can be obtained from the latter by a sequence of such transfers. With multidimensional intervals being defined using vector dominance, an inframodular transfer can be thought of as transferring mass from both sides of an interval to within it.[12]

6. Axiomatizations of new functional forms

The next five contributions to the symposium provide axiomatizations of new functional forms for the analysis of inequality, risk, and welfare.

Galichon and Henry [15] develop a multivariate extension of the rank-dependent expected utility function (29) introduced by Yaari [45] for univariate risks. To do so, they first extend the concepts of quantiles and comonotonicity to the multivariate framework. A *generalized quantile function* is defined as the solution to a maximum correlation problem with respect to a reference probability distribution μ on Ω. Two multivariate distributions X and Y are *μ-comonotonic* if there exists a vector $\omega \in \Omega$ distributed according to μ such that X and Y can be simultaneously rearranged so as to achieve maximal correlation with ω. Galichon and Henry's multivariate generalization of Yaari's functional form evaluates distributions using a weighted sum of generalized quantiles. This objective function satisfies natural multivariate extensions of first-order stochastic dominance and comonotonic independence using the generalized quantiles and comonotonicity concept defined using the reference distribution. Galichon and Henry also characterize risk aversion in this framework and show that the reference distribution can be interpreted as being equilibrium prices.

Surprisingly little attention has been devoted to the measurement of inequality when incomes are uncertain. Notable exceptions are Ben-Porath et al. [4] and Gajdos and Maurin [14]. In a framework in which incomes are state-contingent, Chew and Sagi [9] introduce and axiomatically characterize a class of social welfare functions that is a one-parameter extension of the class of inequality-averse generalized Gini equally-distributed-equivalent income functions applied to the distribution of mean incomes. The parameter ϕ is in $[0, 1)$. The generalized Ginis are obtained when $\phi = 0$. When $\phi > 0$, there is a covariance term that results in the social welfare function favoring uncertain income distributions for which, on average, an individual's income share is positively correlated with a measure of the other individuals' incomes. This term captures a concern for ex post fairness, what Chew and Sagi call a preference for *shared destinies*. Their social welfare functions also exhibit a preference for ex ante fairness and an aversion to aggregate risk. Chew and Sagi also propose a way of a defining an equally-distributed-equivalent income for uncertain income distributions and use it in combination with the procedure described in (25) to identify the class of inequality indices that corresponds to their welfare measures.

Fleurbaey [12] has recently proposed a novel criterion for socially evaluating state-contingent alternatives. His proposal employs the concept of an equally-distributed-equivalent income, but applied to ex post utilities, not incomes. Specifically, his *expected equally-distributed-equivalent* social welfare function evaluates risky social alternatives by first computing the ex post utility u_s in state s that would result in the same level of ex post social welfare as the actual ex post distribution of utilities if everybody had utility u_s and then taking a weighted sum of these values for each state using the probabilities of the states as weights.

Grant et al. [17] provide an axiomatic characterization of this social welfare function using a variant of the framework employed by Harsanyi [22] in his impartial observer theorem. An observer who is stripped of knowledge about his identity has a preference over *product lotteries*,

[12] For two d-vectors ω' and ω'' with $\omega' \leqslant \omega''$, the hyperbox defined above whose vertices are obtained by taking all combinations of the components of these two vectors is the *multidimensional interval* between ω' and ω''.

each of which consists of a lottery over possible personal identities and a lottery over possible outcomes. The axioms in the characterization theorem specify properties of this preference. The social welfare function employs a transform that converts any individual's ex post utility into the ex post utility of the observer. Grant et al. show that the concavity of the transform corresponds to a particular concept of ex post inequality aversion.

The most common social welfare function for evaluating consumption (or income) streams in an infinite horizon model is *discounted utilitarianism*, defined by setting

$$W(c_1, c_2, \ldots) = \sum_{i \in \mathbb{N}} \beta^{t-1} u(c_t), \tag{32}$$

where c_t is the consumption of generation t, u is the utility function used by the social planner, and $\beta \in (0, 1)$ is the utility discount factor. This criterion has been criticized because it may entail that earlier generations make large sacrifices for later ones that are significantly better off. However, when there are nonrenewable resources, it may be the distant future generations who are the ones who are worse off.

In order to overcome these kinds of intergenerational inequity, Zuber and Asheim [46] propose that the discounting in (32) be applied to ranks in the consumption (or, equivalently, utility) stream, not dates. For consumption streams that can be rearranged in a nondecreasing order, by reinterpreting t as the consumption rank (starting with the lowest consumption), (32) is a *rank-discounted utilitarian* social welfare function. Such a function is an infinite-dimensional extension of the generalized Gini functional form (28), with β^{t-1} being the weight attached the tth lowest utility. Zuber and Asheim axiomatically characterize the class of such functions on the domain of consumption streams that can be nondecreasingly reordered. Significantly, discounted rank-discounted utilitarianism not only treats generations equally, it also satisfies a strong form of the Pareto principle, thereby showing the compatibility of criteria on this restricted domain that are incompatible on most domains considered in the literature on evaluating infinite consumption streams. Zuber and Asheim show how to extend their welfare criterion to the set of all consumption streams and axiomatize the resulting class of *extended rank-discounted utilitarian* social welfare functions. They also identify the restrictions required for such a function to be inequality averse and show how the choice of discount rate is related to inequality aversion. The applicability of their approach is illustrated by investigating the properties of optimal policies in two standard growth models.

Sprumont [40] is concerned with the construction of an inequality-averse social welfare ordering over allocations of commodities to individuals. For the most part, the social welfare orderings (or functions) used to analyze issues related to inequality assume that the same utility function is used to evaluate each person's bundle. However, when there is more than one good, one would expect individuals to have different preferences, in which case, it may seem natural to respect these preferences when evaluating the social alternatives. Unfortunately, as Fleurbaey and Trannoy [13] have shown, for some patterns of preferences, the Pareto principle can conflict with a multivariate generalization of the Pigou–Dalton transfer principle that regards a transfer of commodities from someone who has more of every commodity to someone who has less to be welfare improving.

Sprumont [40] argues that there are good reasons why individual preferences should not be respected (e.g., they may be based on incorrect beliefs). However, he thinks that it is reasonable to respect a weaker form of the Pareto principle in which one allocation is socially preferred to a second if everybody agrees that each person's bundle in the former is better than in the latter, a property that he calls *consensus*. Sprumont introduces and axiomatically characterizes a class of social welfare orderings that satisfy this principle and a strong form of *dominance aversion* that regards a reduction in the vector dominance of one individual's bundle by another as a social improvement. The social orderings in this class use a leximin procedure to extend the preferences over commodity bundles on which there is consensus to an ordering over allocations.

7. Inequality aversion and risk aversion

Inequality aversion and risk aversion play a prominent role in many of the articles that have been discussed in the preceding two sections. These issues are the focus of the analysis in the final two contributions to the symposium.

Rothschild and Stiglitz [37] used the criteria for comparing risk discussed in Rothschild and Stiglitz [36] to derive a number of comparative static results that sign the change in some economic variable (such as the amount an individual saves) in response to an increase in risk. In a similar vein, Bommier et al. [5] introduce a general model-free way of defining comparative risk aversion and then use this definition to derive comparative static results on the impact of increased risk aversion on savings behavior in a number of models of risky intertemporal choice. They also use their definition to identify which of the standard classes of utility functions used in the literature on risky intertemporal choice are well ordered in terms of risk aversion.

Bommier et al. restrict attention to preferences that exhibit different risk attitudes but coincide when comparing deterministic outcomes. Their definition of comparative risk aversion begins with a partial order of a set of lotteries interpreted as meaning "weakly riskier than." In their theorems, this partial order is required to be consistent with a (coarser) partial order that regards one lottery to be riskier than a second if it is more "spread out" than a second in a precise sense. One preference is *more risk averse* than a second if for any pair of lotteries ℓ' and ℓ'', ℓ' is weakly preferred to ℓ'' by the second preference whenever ℓ' is weakly preferred to ℓ'' by the first preference and ℓ' is weakly riskier than ℓ''. This definition generalizes the one introduced by Yaari [44] in which the comparison lotteries ℓ'' are deterministic; that is, a preference exhibits greater risk aversion if it has smaller certainty equivalents.

Chambers [8] investigates the relationship between inequality aversion and risk aversion in a model of household decision-making. The individuals in the household have different attitudes towards risk, but share common beliefs. An individual's utility is measured by his certainty-equivalent income and the household welfare function W is a function of these utilities. The household also has a utility function U^W defined over state-contingent aggregate household incomes. For each such bundle, the value of U^W is the value of W that is achieved when the aggregate income in each state is allocated to the individuals so as to maximize W. Chambers regards one household welfare function as being *more inequality averse* than a second if any deviation from equality of utility across individuals that the former prefers is also preferred by the latter. Similarly, as in Yaari [44], one household utility function is regarded as being *more risk averse* than a second if any deviation from equality of aggregate incomes across states that the former prefers is also preferred by the latter. In his main theorem, Chambers shows that the more inequality averse the household welfare function is, the more risk averse the household utility function, thereby establishing a link between inequality aversion and risk aversion in this framework.

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