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Shared destinies and the measurement of social risk equity

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Abstract The evaluation of social risk equity for alternative probability distributions over the potential sets of fatalities is analyzed axiomatically. Fishburn and Straffin (Equity considerations in public risks valuation, Operations Research 37:229–239, 1989) have identified a necessary and sufficient condition for two social risk distributions to be judged to be socially indifferent whenever their associated distributions of risk of death for individuals and for the number of fatalities are the same. It is argued that this approach does not permit society to exhibit any concern for the number of people an individual perishes with. A weakening of the Fishburn–Straffin condition that is compatible with a concern for shared destinies is proposed.

Keywords Social risk evaluation · Social risk equity · Public risk · Shared destinies

1 Introduction

Societies face risks and design public policies to manage these risks. In many cases, these risks involve the potential loss of life. Examples of such risks include hurricanes, earthquakes, epidemics, terrorist attacks, nuclear disasters, and the collapse of bridges. In order to decide which among the feasible risk-reducing policies to implement, it is necessary to

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evaluate these risks from a social point of view. For risks of death, common ways of evaluating the success of a policy are to compute the expected number of lives saved or the reduction in the average risk of death. Such measures fail to account for some important dimensions of risk evaluation, such as the equity of the resulting distribution of risks across individuals or social risk attitudes about the potential number of fatalities. For example, the construction of a levee that ensures that one neighbourhood in a city is safe from floods while leaving the rest of the city unprotected may be regarded as being socially inferior to the construction of a more extensive system of levees that reduces, but does not eliminate, the risk of flooding in all parts of the city. Society may also prefer to have a non-nuclear military defence strategy rather than to acquire nuclear weapons, even if the expected loss of life is the same in both cases, so as to avoid the possibility of a catastrophic loss of life with the latter strategy.

Beginning with Keeney (1980a, 1980b, 1980c), an extensive theoretical literature has developed that is devoted to the analysis of social risk equity. See, for example, Broome (1982), Fishburn (1984), Fishburn and Sarin (1991), Fishburn and Straffin (1989), Harvey (1985), Keeney and Winkler (1985), and Sarin (1985). Attitudes towards social risk equity have also been investigated using surveys and interviews by Keller and Sarin (1988) and Bian and Keller (1999).[1]

One issue that has been considered in this literature concerns the extent to which a satisfactory index of overall social risk equity can be obtained by aggregating indices that measure different dimensions of social risk.[2] Suppose that we are concerned with the risk of death. Each subgroup of the population is a potential fatality set; i.e., each subgroup could be the set of individuals who perish as a result of their exposure to the risk being considered. A social risk distribution is a probability distribution over these fatality sets. Given such a probability distribution, one can compute each individual's probability of death and the probability that there will be any particular number of fatalities. Using this information, Keeney and Winkler (1985), Sarin (1985), and Fishburn and Sarin (1991) have constructed indices of social risk equity for the distributions of individual likelihoods of death and indices of social risk equity for the distributions on the number of fatalities which are then aggregated into an overall index of social risk equity.[3] This two-stage procedure would be justified if two social risk distributions are judged to be socially indifferent whenever their associated distributions of risk for individuals and for the number of fatalities are the same. Assuming that the overall index is continuous, Fishburn and Straffin (1989) have identified a necessary and sufficient condition for this to be the case.

This approach fails to take account of an important dimension of social risk equity in any significant way—a concern for shared destinies, or what is sometimes called "common fates". Societies often care about the number of people who share the same destiny. For example, based on the belief that death is easier to face when one is not alone, one could argue that it is better to have troops patrol a danger area in groups with the risk that they all

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[1] There is also a literature that considers both the benefits and risks of different public policies. See, for example, Fishburn and Sarin (1994) and Keller and Sarin (1995) for discussions of the fairness of different social decision procedures for handling both benefits and risks.

[2] For a thoughtful discussion of some of the other issues that arise when evaluating social risks, see Fleurbaey (2006).

[3] The measures of risks for individuals and for fatalities are sometimes referred to as being utility functions for *ex ante* and *ex post* risk equity, respectively. Both Keeney and Winkler (1985) and Sarin (1985) regard the measure of social risk equity for individuals as being an index of *ex ante* social risk equity and the measure of social risk equity for fatalities as being the sum of an index of *ex post* social risk equity and an index that captures the direct concern for the number of fatalities.

die together rather than conduct separate patrols, even if in both options every soldier faces the same risk of death and the probability of any particular number of fatalities is the same. Furthermore, for any given number of fatalities, society may prefer a more equal to a less equal distribution of these risks. For example, if there is a probability p that k individuals will die, it is better for two groups of size k to all die with probability $\frac{p}{2}$ than to just have one group of size k perish with probability p.

As we shall show, it is possible for the distributions over how many other people one dies with to differ between two social risk distributions even though they share the same distributions of risk of death for individuals and the same distributions of risk for the number of fatalities. Because it is not possible for society to exhibit any concern for shared destinies in such circumstances if one adopts Fishburn and Straffin's condition, we propose a weakening of their condition that does. We prove that our condition implies that any social risk evaluation that is a continuous ordering of the possible social risk distributions only depends on the probabilities that individuals die in a fatality set consisting of k individuals for all possible values of k. Our approach does not preclude taking account of the distributions of risk for individuals and for fatalities because these distributions can be computed using the information permitted by our condition.

As documented by Bian and Keller (1999) and others, attitudes towards risk equity may be culture specific. As a consequence, different societies may have different views about the shared destinies aspect of social risk equity. Our approach is flexible enough to handle divergent views about this issue.

In Sect. 2, we present our formal model of social risk equity evaluation. We consider some examples that illustrate the importance of the three dimensions of social risk equity that we have identified in Sect. 3. In Sect. 4, after first discussing the Fishburn–Straffin condition, we show how this condition can be weakened so as to be compatible with a social concern for shared destinies and we identify the restrictions on the social risk evaluation implied by our axioms. In Sect. 5, we construct social risk evaluation functions that are decomposable into separate indices, each of which measures one of the three dimensions of social risk equity described above. We offer some concluding remarks in Sect. 6.

2 A formal model of social risk equity evaluation

Let $N = \{1, \ldots, n\}$ be a society consisting of $n \geq 2$ individuals. This society faces a social risk. The relevant outcomes for risk evaluation are the potential subsets $S \subseteq N$ of individuals who might die as a consequence of their exposure to this risk. Thus, there are 2^n mutually exclusive outcomes, from $\varnothing$ (nobody dies) to N (everybody dies). Each set S is a *fatality set*. There is uncertainty as to which of these outcomes will be realized, which is captured by a probability distribution p on 2^n. We refer to p as a *social risk distribution*. Let $\mathcal{P}$ denote the set of all such probability distributions.

Following most of the literature on the evaluation of social risk equity, we restrict attention to homogeneous societies. A society is *homogeneous* if any differences between individuals other than their differing exposures to social risks are irrelevant for the social risk assessment. As Fishburn and Straffin (1989) and Fishburn and Sarin (1991) have noted, this assumption is not appropriate if individuals can be partitioned into homogeneous subgroups that differ from one another in a socially relevant characteristic. For example, individuals may be grouped into families or neighbourhoods, with society exhibiting a social preference for dispersing the risks across a number of these groups rather than concentrating them in a small number of them. Fishburn and Sarin (1991) have developed measures of social risk that incorporate a concern for this kind of dispersive equity.

Let $N^* = N \cup \{0\}$. N^* is the number of individuals who might die as a result of the social risk. For all $k \in N^*$, let $\mathcal{T}(k) = \{S \in 2^n || S| = k\}$ be the subsets of the population in which exactly k individuals die. For all $(k, i) \in N \times N$, $\mathcal{S}(k, i) = \{S \in \mathcal{T}(k)|i \in S\}$. $\mathcal{S}(k, i)$ is the set of population subgroups in which person i dies with $k - 1$ other individuals. For the social risk distribution p, the probability that the outcome is a member of $\mathcal{S}(k, i)$ is $\sum_{S \in \mathcal{S}(k,i)} p(S)$.

Let $\mathcal{A}$ denote the set of n-vectors whose components are nonnegative and sum to no more than 1 and $\mathcal{B}$ denote the set of probability distributions over N^*. For all $p \in \mathcal{P}$, let α_p be the vector in $\mathcal{A}$ whose ith component is $\alpha_p(i) = \sum_{S \ni i} p(S)$. That is, $\alpha_p(i)$ is the *ex ante* probability that person i will die. The vector α_p is the *risk profile for individuals*. For all $p \in \mathcal{P}$, let β_p be the probability distribution in $\mathcal{B}$ whose kth component is $\beta_p(k) = \sum_{S \in \mathcal{T}(k)} p(S)$. That is, $\beta_p(k)$ is the *ex ante* probability that there will be exactly k fatalities. The probability distribution β_p is the *risk profile for fatalities*.

For all $p \in \mathcal{P}$, let M_p be the $n \times n$ matrix whose entry in the kth row and ith column is $M_p(k, i) = \sum_{S \in \mathcal{S}(k,i)} p(S)$. That is, $M_p(k, i)$ is the probability that person i dies with exactly $k - 1$ other individuals. For $k = 1, \ldots, n$, the kth row of M_p is $M_{pk} = (M_p(k, 1), \ldots, M_p(k, n))$. Note that $\alpha_p(i) = \sum_{k=1}^{n} M_p(k, i)$ for all $i \in N$, $\beta_p(k) = \frac{1}{k} \sum_{i=1}^{n} M_p(k, i)$ for all $k \in N$, and $\beta_p(0) = 1 - \sum_{k=1}^{n} \beta_p(k)$. Thus, knowledge of M_p is sufficient to compute both α_p and β_p.

A government (or other body) can undertake policies that affect the social risk distribution. In order to determine what policy to implement, it needs to know how to rank different social risk distributions in $\mathcal{P}$ in terms of their social acceptability. This ranking is described by a binary relation $\succeq$ on $\mathcal{P}$ which is interpreted as meaning "weakly socially preferred to". Let $\sim$ and $\succ$ denote the symmetric and asymmetric factors of $\succeq$, respectively. Henceforth, we refer to $\succeq$ as the *social risk equity evaluation*. Note that the restriction of $\succeq$ to social risk distributions in which there is one fatality set that occurs with probability 1 is a social preference on the set of possible *ex post* consequences.

There are two basic properties that we require $\succeq$ to satisfy. Our first restriction on $\succeq$ is that it is an *ordering* of $\mathcal{P}$.

Axiom 1 *$\succeq$ is a reflexive, complete, and transitive binary relation on $\mathcal{P}$.*

We also require $\succeq$ to be *continuous*.

Axiom 2 *The sets $\{q \in \mathcal{P}|p \succ q\}$ and $\{q \in \mathcal{P}|q \succ p\}$ are open for all $p \in \mathcal{P}$.*

3 Some examples

As we have discussed, one approach to constructing the social risk equity evaluation $\succeq$ (or a social utility function representing this relation) is to separately construct measures of social equity for the risk profiles for individuals and for the risk profiles for the number of fatalities, and then to aggregate these measures into an overall assessment of social risk equity. With this approach, all of the relevant information about a probability distribution $p \in \mathcal{P}$ is contained in the corresponding distributions α_p and β_p. In effect, this approach regards social risk equity as being composed of two dimensions, with overall social risk equity being decomposable into these two components.

Such a decomposition neglects information about the distribution p that may be essential for evaluating social risks, such as information about shared destinies. It is therefore important not to assume *a priori* that a measure of social risk equity is decomposable. In other words, a more holistic approach is warranted, and this is provided by the social risk equity evaluation $\succeq$. Nevertheless, how society takes account of the risks that individuals face and of the possible number of fatalities plays an important role in forming an overall evaluation of social risk, even if these two dimensions of social risk do not capture all that is of social concern. The following examples illustrate the importance of these considerations.

Example 1 The set of individuals who live in the same country is $N = \{1, \dots, 100\}$. Everybody lives in either city A or city B, both of which have the same number of residents. For concreteness, suppose that $N_A = \{1, \dots, 50\}$ live in city A and $N_B = \{51, \dots, 100\}$ live in city B. There is a probability x that a hurricane will strike this country, but there is uncertainty as to its path. The hurricane will destroy city A with probability $\frac{x}{2}$ and with the same probability it will destroy city B. In each case, all of the inhabitants of the affected city will die and all of the residents of the other city are spared.

Given its limited budget for emergency responses and the time available for hurricane preparation, the government has a choice between three options:

Option 1. Devote all of the emergency aid budget to the protection of city A and the evacuation of its residents, which reduces the risks faced by residents of city A to 0, but does not change the risks faced by residents of city B.

Option 2. Devote all of the emergency aid budget to the protection of city B and the evacuation of its residents, which reduces the risks faced by residents of city B to 0, but does not change the risks faced by residents of city A.

Option 3. Share equally the emergency aid budget between cities A and B, which reduces the risk for each resident of the country to $\frac{x}{4}$.

Let p^j be the social risk distribution corresponding to option j, $j = 1, 2, 3$. These three distributions are summarized in the following table. Of course, $p^j(S) = 0$ for all subgroups S not shown.

	$\varnothing$	N_A	N_B
p^1	$1 - \frac{x}{2}$	0	$\frac{x}{2}$
p^2	$1 - \frac{x}{2}$	$\frac{x}{2}$	0
p^3	$1 - \frac{x}{2}$	$\frac{x}{4}$	$\frac{x}{4}$

It can easily be checked that the risk profiles for the number of fatalities associated with these three options are as given in the following table.

	0	1	$\cdots$	49	50	51	$\cdots$	100
β_{p^1}	$1 - \frac{x}{2}$	0	$\cdots$	0	$\frac{x}{2}$	0	$\cdots$	0
β_{p^2}	$1 - \frac{x}{2}$	0	$\cdots$	0	$\frac{x}{2}$	0	$\cdots$	0
β_{p^3}	$1 - \frac{x}{2}$	0	$\cdots$	0	$\frac{x}{2}$	0	$\cdots$	0

Thus, all three options result in the same probability distribution over the number of fatalities: with probability $1 - \frac{x}{2}$, nobody will die, and with probability $\frac{x}{2}$, half the population will die. As a consequence, the expected number of deaths is the same whatever option is chosen.

The risk profiles for individuals associated with these three options are presented in the following table.

	1	2	$\cdots$	50	51	52	$\cdots$	100
α_{p^1}	0	0	$\cdots$	0	$\frac{x}{2}$	$\frac{x}{2}$	$\cdots$	$\frac{x}{2}$
α_{p^2}	$\frac{x}{2}$	$\frac{x}{2}$	$\cdots$	$\frac{x}{2}$	0	0	$\cdots$	0
α_{p^3}	$\frac{x}{4}$	$\frac{x}{4}$	$\cdots$	$\frac{x}{4}$	$\frac{x}{4}$	$\frac{x}{4}$	$\cdots$	$\frac{x}{4}$

If options 1 or 2 are chosen, half the population has an individual probability of dying equal to $\frac{x}{2}$, whereas the other half has an individual probability of dying equal to 0. However, if option 3 is adopted, then each individual has the same probability of dying, namely, $\frac{x}{4}$.

In this example, the three options affect how the risk of dying is distributed between individuals, but they do not differ in the probability that any given number of individuals will die. In effect, the government must simply decide how best to distribute the probability $\frac{x}{2}$ that half of its residents will die among all of its residents given the constraint that individuals who live in the same city must share the same destiny. On equity grounds, option 3 is socially preferred to options 1 and 2 because the individual probabilities of death are more equally distributed if option 3 is chosen. Because the society is homogeneous, it is reasonable to regard options 1 and 2 as being socially indifferent.

In general, the distribution of individual probabilities of dying is clearly not sufficient to evaluate social risk equity because it does not take into account the fairness of the resulting probability distribution over the number of fatalities. The following example illustrates a situation in which such considerations are decisive.

Example 2 The set of passengers on a boat is $N = \{1, 2, 3, 4\}$. The boat is sinking, but there is only one lifeboat and it is designed for only two people. The skipper (whose code of honour requires him to go down with his boat) has the choice between the two following options:

Option 1. Let exactly half of the passengers board the lifeboat. These passengers will survive for sure, whereas the passengers who remain on the sinking boat will die for sure. In order to choose who boards the lifeboat, the skipper designs a fair lottery. Thus, each passenger has a probability $\frac{1}{2}$ of obtaining a place on the lifeboat.

Option 2. Let all of the passengers board the lifeboat. The overcrowded lifeboat will sink with probability $\frac{1}{2}$, leading to the death of all of the passengers. If the lifeboat does not sink, all of the passengers will survive.

Let q^j be the social risk distribution corresponding to option j, $j = 1, 2$. These distributions are summarized in the following table, where $q^j(S) = 0$ for any subgroup S not shown.

	$\varnothing$	$\{1, 2\}$	$\{1, 3\}$	$\{1, 4\}$	$\{2, 3\}$	$\{2, 4\}$	$\{3, 4\}$	$\{1, 2, 3, 4\}$
q^1	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	0
q^2	$\frac{1}{2}$	0	0	0	0	0	0	$\frac{1}{2}$

As illustrated in the following table, every individual has a probability of death equal to $\frac{1}{2}$ regardless of the option chosen.

	1	2	3	4
α_{q^1}	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
α_{q^2}	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

The probability distributions over the number of fatalities are presented in the following table.

	0	1	2	3	4
β_{q^1}	0	0	1	0	0
β_{q^2}	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$

Although the expected number of deaths is the same whatever option is chosen (namely, half the number of passengers), the two options lead to very different distributions over the number of deaths. If option 1 is chosen, half the passengers will die for sure, whereas if option 2 is chosen, with probability $\frac{1}{2}$ all of the passengers will survive and with the same probability the lifeboat will sink and they all die.

In this example, it is reasonable to suppose that attitudes about the actual number of fatalities should be decisive. If one believes that society should have a preference for avoiding catastrophes, then option 1 is socially preferred to option 2. If one instead thinks that it is more equitable if individuals share a common fate, then option 2 is socially preferred to option 1. However, if society expresses no concern for common fate considerations, then only the expected number of fatalities should matter in this example, with the consequence that options 1 and 2 are socially indifferent.[4]

The preceding examples have been designed to illustrate circumstances in which either only the risk profiles for individuals or the risk profiles for fatalities are relevant for the social risk evaluation. A more complex example is needed in order to illustrate the importance of the shared destinies considerations described in the Introduction. In the following example, the risk profiles for individuals and for fatalities are identical in the three available options. However, by assuming that there is a social preference for spreading the risk of a given number of individuals dying more equally over individuals and that there is a strong social aversion to individuals facing the risk of dying alone, it is possible to strictly rank the three alternatives.

Example 3 The set of workers at a factory is $N = \{1, 2, 3, 4, 5\}$. The working day is partitioned into four shifts, with each employee working two consecutive shifts. The workers have different skills, so it is not possible to arbitrarily assign them to shifts. There are three feasible options:

Option 1. Individual 1 works in shifts 1 and 2, individual 2 works in shift 3, individual 3 works in shift 4, and individuals 4 and 5 work in shifts 3 and 4.

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[4] Keeney (1980a) has investigated the implications of having society exhibit a preference for catastrophe avoidance. Fishburn (1984) and Fishburn and Straffin (1989) have formulated axioms for social risk evaluation that distinguish between the three attitudes towards common fates described above. Bommier and Zuber (2008) have recently argued that a preference for catastrophe avoidance can provide an ethical foundation for social discounting.

Option 2. Individual 1 works in shifts 1 and 2, individual 2 works in shift 3, individual 3 works in shift 4, and individuals 4 and 5 work in shifts 1 and 2.

Option 3. Individual 1 works in shifts 1 and 2, individual 2 works in shift 3, individual 3 works in shift 4, and individuals 4 and 5 work in shifts 2 and 3.

Note that these three options only differ in the shifts assigned to individuals 4 and 5. In each shift, with probability $\frac{1}{8}$, there is an accident which kills all of the workers present.

Let r^j be the social risk distribution corresponding to option j, $j = 1, 2, 3$. For subgroups S for which the probability of dying is positive in at least one of the three options, the values of $r^j(S)$ are shown in the following table.

	$\varnothing$	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,4,5\}$	$\{2,4,5\}$	$\{3,4,5\}$
r^1	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0	$\frac{1}{8}$	$\frac{1}{8}$
r^2	$\frac{1}{2}$	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	0	0
r^3	$\frac{1}{2}$	$\frac{1}{8}$	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	0

As shown in the following table, the risk profiles for these individuals are identical in the three options.

	1	2	3	4	5
α_{r^1}	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$
α_{r^2}	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$
α_{r^3}	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$

In addition, as shown in the following table, the risk profiles for fatalities are also the same in these options.

	0	1	2	3	4	5
β_{r^1}	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$	0	0
β_{r^2}	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$	0	0
β_{r^3}	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$	0	0

If the risk profiles for individuals and for the number of fatalities were all that mattered for the evaluation of social risk equity, then the three options would be regarded as being socially indifferent. However, if society exhibits a concern for shared destinies, then this conclusion need not hold. To understand why, we analyze the matrices M_{r^j} associated with the three social risk distributions. Recall that the generic element $M_{r^j}(k, i)$ denotes the probability of individual i dying in a group of size k with the distribution r^j. In our example:

$$M_{r^1} = \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{8} & \frac{1}{8} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{r^2} = \begin{bmatrix} 0 & \frac{1}{8} & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$M_{r^3} = \begin{bmatrix} \frac{1}{8} & 0 & \frac{1}{8} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \frac{1}{8} & \frac{1}{8} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Note that these matrices only differ in the submatrices obtained by deleting the last two rows and columns.

It seems reasonable to suppose that for a given probability of a fixed number of people dying, it is better if this risk is spread more evenly over the individuals, provided that the risk profiles for individuals and for fatalities are unaffected. In this example, modulo a permutation of identities, r^2 and r^3 only differ in the distribution across individuals of the probability of dying with two other individuals, i.e., in $M_{r^2 3}$ and $M_{r^3 3}$. Because $M_{r^3 3}$ is less unequal than $M_{r^3 2}$, it follows that r^3 is socially preferred to r^2, at least if the identities of who dies is not a matter of social concern. Similarly, r^3 is socially preferred to r^1 because the probability of dying alone is spread more equally in r^3.

The comparison of r^1 and r^2 is more complex. The risk of dying alone is spread more equally with r^2, whereas the risk of dying with two other individuals is spread more evenly with r^1. If society expresses a strong aversion to individuals facing the risk of dying alone, this might be enough to tip the balance in favour of r^2, at least if the risk-spreading benefits of r^1 are not too large, because there is the same probability that someone dies alone in both options, but more people face this risk with r^2. If, in fact, $r^2 \succ r^1$, then we have $r^3 \succ r^2 \succ r^1$.

When, as in the comparison of r^1 and r^2, risk-spreading considerations for different values of k conflict, how they are resolved could depend on the values of k for which this issue arises and on how substantial the risk-spreading benefits are. For example, society might well be prepared to accept a more unequal distribution when a relatively small number of individuals will die in order to avoid a very unequal distribution should many more individuals perish. We remain agnostic about such trade-offs. Nevertheless, our example suggests that basing a social risk evaluation solely on the risk distributions for individuals and for fatalities is inadequate in some circumstances.

4 The new axiom and characterization theorem

In this section, we propose a restriction on the social risk evaluation $\succeq$ that, when combined with Axioms 1 and 2, permits one to take account of the number of people who share one's fate, not just the risk profiles for individuals and the risk profiles for fatalities. Our new condition is a weakened version of an axiom introduced by Fishburn and Straffin (1989). After reviewing Fishburn and Straffin's contribution, we present our new condition and then identify the restrictions on the social risk evaluation implied by our axioms.

4.1 Simply related social risk distributions

Fishburn and Straffin were interested in identifying restrictions on the social risk evaluation that are necessary and sufficient for the overall evaluation to only depend on the risk profiles for individuals and the risk profiles for fatalities. To state Fishburn and Staffin's axiom, we first need to introduce the concept of *simply related* social risk distributions.

Definition 1 Two social risk distributions $p, q \in \mathcal{P}$ are *simply related* if either (i) $p = q$ or (ii) $p \neq q$ and there exist $A, B, C, D \subseteq N$ and $\delta > 0$ such that $A \neq \varnothing$, $A \cap B = \varnothing$, $|A| = |B|$, $(C \cup D) \cap (A \cup B) = \varnothing$, $C \cup D \neq \varnothing$, and p and q are identical except that:

1. $q(A \cup C) = p(A \cup C) - \delta$
2. $q(B \cup C) = p(B \cup C) + \delta$
3. $q(A \cup D) = p(A \cup D) + \delta$
4. $q(B \cup D) = p(B \cup D) - \delta$.

Consider the case in which the social risk distributions p and q are not identical. The groups A and B in this definition are disjoint, but contain the same number of individuals. These two groups are also disjoint from both C and D. The distributions p and q are identical except for how likely groups A and B are to perish with either C or D. In the case of A, in moving from p to q, some probability is shifted from dying with the members of C to dying with the members of D. The reverse shift of the same amount of probability applies to B. Thus, p and q only differ in the relative likelihood that members of A and B die with members of C and D. It is easily checked that if p and q are simply related, then $(\alpha_p, \beta_p) = (\alpha_q, \beta_q)$. That is, the risk profiles for individuals and the risk profiles for fatalities associated with p and q are identical.

For homogeneous societies, Fishburn and Straffin (1989) regard two social risk distributions as being socially indifferent if they are simply related.

Axiom 3 *For all $p, q \in \mathcal{P}$, $p \sim q$ if p and q are simply related.*

Fishburn and Strafin (1989, Theorem 1) have established the following theorem.

Theorem 1 *Assume that Axioms 1 and 2 hold. Then Axiom 3 holds if and only if for all $p, q \in \mathcal{P}$,*

$$(\alpha_p, \beta_p) = (\alpha_q, \beta_q) \Rightarrow p \sim q.$$

Thus, given Axioms 1, 2, and 3, the only information that is needed to determine how to socially rank p and q in terms of social risk equity are the corresponding risk profiles for individuals and for fatalities. For the reasons discussed earlier, Fishburn and Straffin do not advocate the adoption of Axiom 3 in heterogeneous societies. However, they suggest that if the composition of the set of individuals who die is not of social concern, then this axiom is an appealing restriction on the social risk evaluation $\succeq$. But even if one believes that individuals should be treated symmetrically, Axiom 3 is incompatible with taking account of how many people an individual shares his fate with provided that $\succeq$ is transitive.

To illustrate why this is the case, we reconsider Example 3. In this example, the distributions r^1 and r^3 are simply related, with $A = \{1\}$, $B = \{3\}$, $C = \varnothing$, $D = \{4, 5\}$, and $\delta = \frac{1}{8}$. Similarly, r^3 and r^2 are simply related, with $A = \{1\}$, $B = \{2\}$, $C = \varnothing$, $D = \{4, 5\}$ and $\delta = \frac{1}{8}$. Therefore, Axiom 3 together with the transitivity of $\succeq$ imply that $r_1 \sim r_2 \sim r_3$. However, in our discussion of Example 3, we have argued that this social preference is unacceptable if the number of people one dies with and the distribution of the risks of dying conditional on the number of fatalities are matters of social concern.

4.2 Strongly related social risk distributions

In the definition of simply related social risk distributions, the groups A and B are of the same size, but C and D may not be. As a consequence, the redistribution of risk need not be among groups of the same size. It is for this reason that the social risk evaluation in the Fishburn–Straffin approach is insensitive to the number of individuals one dies with. To allow for the social risk evaluation to take account of shared destiny considerations, we propose to weaken Axiom 3 so that it only applies to simply related social risk distributions for which the redistribution of risk is made between same-sized groups. Such distributions are called *strongly related*.

Definition 2 Two social risk distributions $p, q \in \mathcal{P}$ are *strongly related* if either (i) $p = q$ or (ii) $p \neq q$ and there exist $A, B, C, D \subseteq N$ and $\delta > 0$ such that $A \neq \varnothing$, $A \cap B = \varnothing$, $|A| = |B|$, $|C| = |D|$, $(C \cup D) \cap (A \cup B) = \varnothing$, $C \cup D \neq \varnothing$, and p and q are identical except that:

1. $q(A \cup C) = p(A \cup C) - \delta$
2. $q(B \cup C) = p(B \cup C) + \delta$
3. $q(A \cup D) = p(A \cup D) + \delta$
4. $q(B \cup D) = p(B \cup D) - \delta$.

Clearly, if p and q are strongly related, then they are also simply related. For all $p, q \in \mathcal{P}$, if p and q are strongly related, we say that q is obtained from p by the *strongly related shift* $q - p$. We require two social risk distributions to be socially indifferent if they are strongly related. Thus, it is not a matter of social concern with whom an individual dies with, but it is of social concern how many individuals he shares this fate with.[5]

Axiom 4 *For all $p, q \in \mathcal{P}$, $p \sim q$ if p and q are strongly related.*

By replacing Axiom 3 with Axiom 4 in Theorem 1, we obtain the following theorem.

Theorem 2 *Assume that Axioms 1 and 2 hold. Then Axiom 4 holds if and only if for all $p, q \in \mathcal{P}$,*

$$M_p = M_q \Rightarrow p \sim q. \tag{1}$$

Proof (a) Suppose that (1) holds and that p and q are strongly related. Consider any $i \in N$. There is some group S containing i for which the probability of dying differs in p and q only if $i \in \{A, B, C, D\}$. For such i, because $|A| = |B|$, $|C| = |D|$, and $A \cup B$ is disjoint from $C \cup D$, in going from p to q any loss of (resp. gain in) probability for some group that i is a member of is exactly compensated for by a gain in (resp. loss of) probability for some other group of the same size that also contains i. Hence, $M_p = M_q$. Thus, by (1), $p \sim q$.

(b) We now show that Axioms 1, 2, and 4 are sufficient for (1). For all $p \in \mathcal{P}$ and all $k \in N^*$ such that $\sum_{S \in \mathcal{T}(k)} p(S) \neq 0$, let $\tilde{p}^k \in \mathcal{P}$ be defined by setting

$$\tilde{p}^k(S) = \begin{cases} \dfrac{p(S)}{\sum_{T \in \mathcal{T}(k)} p(T)} & \text{if } S \in \mathcal{T}(k) \\ 0 & \text{if } S \notin \mathcal{T}(k). \end{cases}$$

---

[5] Recall that we are dealing with homogeneous societies. When this is not the case, the identity of who perishes together is of concern. For example, it is both individually and socially desirable that both parents of a child not die together leaving the child an orphan.

If $\sum_{S\in\mathcal{T}(k)} p(S)=0$, let $\tilde{p}^k$ be defined by setting

$$\tilde{p}^k(S)=\begin{cases}1 & \text{if } S=\varnothing\\ 0 & \text{if } S\neq\varnothing.\end{cases}$$

Note that for all $p\in\mathcal{P}$ and all $(k,i)\in N\times N$,

$$M_p(ki)=\left(\sum_{S\ni i}\tilde{p}^k(S)\right)\left(\sum_{T\in\mathcal{T}(k)} p(T)\right)=\alpha_{\tilde{p}^k}(i)\beta_p(k)$$

and

$$\beta_{\tilde{p}^k}(k)=1.$$

Furthermore, for all $p\in\mathcal{P}$,

$$p=\sum_{k=0}^{n}\tilde{p}_k\beta_p(k). \tag{2}$$

Now assume that $p,q\in\mathcal{P}$ are such that $M_p=M_q$ and, hence, that $\alpha_p=\alpha_q$ and $\beta_p=\beta_q$. Assume, furthermore, that $p(S)$ and $q(S)$ are rational for all S. Then, for all $(k,i)\in N\times N$, $\alpha_{\tilde{p}^k}(i)=\alpha_{\tilde{q}^k}(i)$. Thus, by the argument in the proof of Lemma 1 in Fishburn and Straffin (1989), for all $k\in N$, there exists a sequence $r_1^k, r_2^k,\ldots,r_{t(k)}^k$ of probability distributions such that $\tilde{p}^k$ is simply related to r_1^k, r_τ^k is simply related to $r_{\tau+1}^k$ for all $\tau\in\{1,\ldots,t(k)-1\}$, and $r_{t(k)}^k$ is simply related to $\tilde{q}^k$.[6] Because $\tilde{p}^k(S)=0$ for all $S\notin\mathcal{T}(k)\backslash\{\varnothing\}$ and $\tilde{q}^k(S)=0$ for all $S\notin\mathcal{T}(k)\backslash\{\varnothing\}$, all of these distributions are actually strongly related. Let $s_\tau^k=r_{\tau+1}^k-r_\tau^k$ if $\tau\in\{1,\ldots,t(k)-1\}$, $s_0^k=r_1^k-\tilde{p}^k$, and $s_{t(k)}^k=\tilde{q}^k-r_{t(k)}^k$. We thus have, for all $k\in N$,

$$\tilde{q}^k=\tilde{p}^k+\sum_{\tau=0}^{t(k)}s_\tau^k. \tag{3}$$

Equations (2) and (3) imply that

$$\begin{aligned}q&=\sum_{k=0}^{n}\tilde{q}^k\beta_q(k)\\&=\sum_{k=0}^{n}\left(\tilde{p}^k+\sum_{\tau=0}^{t(k)}s_\tau^k\right)\beta_q(k)\\&=\sum_{k=0}^{n}\tilde{p}^k\beta_q(k)+\sum_{k=0}^{n}\sum_{\tau=0}^{t(k)}s_\tau^k\beta_q(k)\\&=p+\sum_{k=0}^{n}\sum_{\tau=0}^{t(k)}s_\tau^k\beta_q(k).\end{aligned}$$

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[6] Note that $\tilde{p}$ and $\tilde{q}$ have zero probability for all nonempty sets of size different from k. Thus, the Claim on p. 236 in Fishburn and Straffin (1989) can only hold with $|G|=k$ and $|H|=k$. This guarantees that the sets C and D in their proof have the same cardinality.

Thus, q is obtained by adding to p a sequence of strongly related shifts. Axiom 4 therefore implies that $p \sim q$.

By using the continuity of $\succeq$ (Axiom 2), this conclusion also holds if any $p(S)$ or $q(S)$ is irrational. For the details of this extension argument, see Fishburn and Straffin (1989). □

5 Decomposable social risk evaluation functions

In order to illustrate the added flexibility provided by our weakening of Axiom 3, in this section, we construct social risk evaluation functions that are decomposable into separate indices that measure the three dimensions of social risk equity that we have considered: shared destinies, individual risks, and fatalities. These functions are then used to evaluate the options in Example 3.

Let $V^1\colon [0,1]^n \to \mathbb{R}$ be a measure of social risk equity for distributions of risk for individuals. For $p \in \mathcal{P}$, the argument of V^1 is α_p, the risk distribution for individuals. Similarly, let $V^2\colon [0,1]^{n+1} \to \mathbb{R}$ be a measure of social risk attitudes towards the number of fatalities. For $p \in \mathcal{P}$, the argument of V^2 is β_p, the risk distribution for fatalities. Specific functional forms for V^1 and V^2 may be found in Keeney and Winkler (1985), Sarin (1985), and Fishburn and Sarin (1991).

For each $p \in \mathcal{P}$ and each $k \in N$, the distribution M_{pk} lists the probabilities that each individual perishes with exactly $k-1$ other individuals. As we have argued, a social risk evaluation should be sensitive to the inequality in these distributions. This sensitivity is captured by a function $U\colon [0,1]^n \to \mathbb{R}$. For $p \in \mathcal{P}$ and $k \in N$, the argument of U is the distribution M_{pk}. We assume that U is strictly Schur-concave and decreasing in each of its arguments. The function U is strictly Schur-concave if it is symmetric in its arguments and if its value increases if there is a rank-preserving transfer of probability of dying with $k-1$ other individuals from one individual to some other individual who has a smaller probability of dying with this many individuals. The latter requirement is simply the familiar Pigou–Dalton transfer principle for income inequality indices (see Sen 1973) applied to the distributions M_{pk}. All else equal, it is better if the probability that anyone dies with a fixed number of individuals is decreased, which is why U is assumed to be decreasing.

For each M_{pk}, the value $U(M_{pk})$ is weighted by $h(k)$, where the function $h\colon N \to \mathbb{R}_{++}$ captures a direct social concern for the number of individuals someone perishes with. Let $\mathcal{M}$ denote the set of all $n \times n$ matrices whose entries are in $[0,1]$. The contribution of shared destiny considerations to our overall measure of social risk equity is given by the function $V^3\colon \mathcal{M} \to \mathbb{R}$, where for each $p \in \mathcal{P}$,

$$V^3(M_p) = \sum_{k=1}^{n} h(k)U(M_{pk}).$$

By adding the functions V^1, V^2, and V^3, we obtain an overall measure of social risk equity. In other words, with this construction, we are supposing that the social risk evaluation $\succeq$ is represented by the social risk evaluation function $W\colon \mathcal{P} \to \mathbb{R}$ given by

$$W(p) = V^1(\alpha_p) + V^2(\beta_p) + V^3(M_p),$$

for all $p \in \mathcal{P}$.

We now use the function W to analyze Example 3. In this example, because $\alpha_{r^1} = \alpha_{r^2} = \alpha_{r^3}$ and $\beta_{r^1} = \beta_{r^2} = \beta_{r^3}$, the three risk distributions being considered are socially ranked by the values of V^3. Because V^3 is additively separable and the matrices M_{r^1}, M_{r^2}, and M_{r^3} only differ in their first and third rows, we only need to consider values for k equal to 1 and 3. Because U is symmetric, $U(M_{kp}) = U(\tilde{M}_{kp})$, where $\tilde{M}_{kp}$ is a nondecreasing permutation of M_{kp}. Thus, we need to determine how U ranks the distributions in the following table, where the column headings denote individuals.

	1	2	3	4	5
$a = \tilde{M}_{r^1 1}$	$\frac{1}{4}$	0	0	0	0
$b = \tilde{M}_{r^1 3} = \tilde{M}_{r^3 3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	0
$c = \tilde{M}_{r^2 1} = \tilde{M}_{r^3 1}$	$\frac{1}{8}$	$\frac{1}{8}$	0	0	0
$d = \tilde{M}_{r^2 3}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	0

Our assumptions on U imply that $U(c) > U(a) > U(b) > U(d)$. The first and third inequalities hold because U is strictly Schur-concave and the second inequality holds because U is decreasing.

First, we compare r^1 and r^3. We have

$$r^1 \succeq r^3 \leftrightarrow h(1)U(a) + h(3)U(b) \geq h(1)U(c) + h(3)U(b).$$

Because $h(1) > 0$ and $U(c) > U(a)$, $r^3 \succ r^1$. As noted in our discussion of Example 3, r^3 is obtained from r^1 by spreading the risk of dying alone more equally among the individuals without affecting the risk distributions for individuals or for fatalities. Thus, a social risk evaluation that is averse to this kind of inequality should rank r^3 above r^1.

Next, we compare r^2 and r^3. We have

$$r^2 \succeq r^3 \leftrightarrow h(1)U(c) + h(3)U(d) \geq h(1)U(c) + h(3)U(b).$$

Because $h(3) > 0$ and $U(b) > U(d)$, $r^3 \succ r^2$. Note that r^3 is obtained from r^2 by spreading the risk of dying with two other individuals more equally among the individuals without affecting the risk distributions for individuals or for fatalities.

Finally, we compare r^1 and r^2. We have

$$\begin{aligned} r^1 \succeq r^2 &\leftrightarrow h(1)U(a) + h(3)U(b) \geq h(1)U(c) + h(3)U(d) \\ &\leftrightarrow h(3)[U(b) - U(d)] \geq h(1)[U(c) - U(a)]. \end{aligned}$$

The terms in square brackets are both positive. If there is a strong social aversion to having individuals die on their own, then $h(3) > h(1)$. As we have previously noted, the risk-spreading considerations for the two values of k that are relevant point in opposite directions. Nevertheless, a clear-cut social ranking of r^1 and r^2 emerges if society is sufficiently averse to having individuals perish on their own provided that the benefits from risk spreading as measured by $U(c) - U(a)$ are not too much greater than those measured by $U(b) - U(d)$.

6 Concluding remarks

There are substantial informational advantages associated with constructing an index of social risk equity if two social risk distributions are judged to be socially indifferent whenever their associated distributions of risk of death for individuals and for the number of fatalities are the same. As noted by Fishburn and Straffin (1989), to characterize a social risk distribution $p \in \mathcal{P}$, $2^n - 1$ quantities are needed, whereas only $2n - 1$ quantities are needed to compute α_p and β_p. The computational complexity of M_p lies between these two extremes. For any $p \in \mathcal{P}$, the entries in the last row of the matrix M_p are identical because everybody has the same probability of dying with $n - 1$ other people. Thus, $n(n - 1) + 1$ quantities are needed to compute M_p. While not as informationally parsimonious as a social risk evaluation that only depends on the distributions of risks for individuals and for fatalities, our approach is less demanding than a social risk evaluation based on all of the information in a social risk distribution because the computation of p is exponential in n, whereas the computation of M_p is only polynomial in n.[7] While such practical considerations are important, they should not be decisive if important aspects of social risk equity are neglected, such as a concern for shared destinies. For this reason, we believe that the ability to take account of this concern in our approach to social risk evaluation outweighs the informational advantages of the approach of Fishburn and Straffin.

We have shown that the social risk evaluation only depends on the probabilities that individuals die in a fatality set with k individuals for $k = 1, \dots, n$ if our three axioms are satisfied. Because the risk distributions for individuals and for fatalities can be computed from this information, a wider class of social risk evaluations are compatible with our axioms than with the axioms used by Fishburn and Straffin (1989). In particular, the social risk evaluation is able to exhibit a concern for the shared destinies of the members of society over and above what is possible from only knowledge of the risk profiles for individuals and for fatalities, which is not possible with the approach of Fishburn and Straffin. Our axioms are consistent with different views on how shared destinies matter.

By considering additional axioms, further structure can be imposed on the social risk evaluation. A major focus of the analysis of Fishburn and Straffin (1989) is the identification of maximally consistent sets of axioms that include Axioms 1, 2, and 3. A natural extension of our analysis would be to undertake a similar exercise for Axioms 1, 2, and 4.

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[7] In practice, many of the entries in M_p could be 0, which mitigates this concern somewhat.

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